# PATENTS, REAL OPTIONS AND FIRM PERFORMANCE*

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Analysing the new IFS-Leverhulme database on over 200 major British firms since 1968 we show that patents have an economically and statistically significant impact on firm-level productivity and market value. While patenting feeds into market values immediately it appears to have a slower effect on productivity. This generates valuable real options because patents provide exclusive rights to develop new innovations, enabling firms to delay investments. Higher market uncertainty, which increases the value of real options, reduces the impact of new patents on productivity. If the government's policy to reduce uncertainty is successful then this should increase the productivity of Britain's knowledge capital.

There is a consensus that technological advance is crucial for economic performance, but measuring technology has always been one of the most perplexing problems facing empirical economics. One tradition, epitomised by Solow (1957), is to measure technology as a residual from a production function. The problem is that this residual also contains the measurement error from the production function estimation, and so provides only an indirect link to productivity. A second tradition, which this paper follows, is to construct observable proxies for technical change. The most popular measure of technology is research and development (R&D) expenditures. Unfortunately at the firm level there was no requirement to report R&D expenditures in Britain before 1989 (even for larger firms), so this hampers the generation of a long time series. Innovation counts have been frequently used in the United Kingdom, but the best series for these ended in 1983 (see Pavitt *et al.*, 1987; Blundell *et al.*, 1999; Geroski, 1990).

Counts of patents have also been a popular choice to proxy innovation. And patents themselves contain a wealth of other information (eg Lerner, 2000). In particular, the front of a patent details other patents which contributed to the knowledge underlying the new patent. This information can be used in a variety of different ways. We start off with the most obvious use. A patent which is cited many times is more likely to be valuable than a patent which is rarely cited (see Griliches, 1990). Other researchers have used patent citations as a 'paper flow' to track the way knowledge spills over between organisations and areas (see Henderson *et al.*, 1993; Jaffe and Trajtenberg, 1998) and this is a route that we are pursuing in complementary work.

We look at the impact of patents on two measures of company performance – productivity and market value. Production functions are more easily interpretable and comparable with other work. Market value is a more forward looking measure,

which has attractions for the analysis of an activity whose pay-off may not be for many years into the future.[1]

From our preliminary work with the data it became apparent that while patents have an immediate impact upon market values they take time to affect productivity. One potential explanation is that the new products and processes which are covered by the patents have to be embodied in new capital equipment and training. Firms may also need to undertake further research and development, as well as expensive marketing and advertising to promote their new products.[2] As such, this will involve extensive sunk cost investments – these capital, training, research and marketing outlays will be (at least partially) irreversible. But since patents provide firms with the exclusive rights to their new technologies they have the option to wait until making these sunk costs investments. When market conditions are uncertain, this will generate valuable real options. Therefore, by giving firms a legally protected right to delay investing, patents provide a test of the importance of real options.

We adapt the developing real options literature to explain the take up of new products and processes covered by patents.[3] The theories developed in this paper predict that higher market uncertainty will lead firms to be more cautious about their investments. We use this theory to then derive empirical predictions on the relationship between patents and uncertainty and empirically test them. This builds on earlier work valuating patents as options by Pakes (1986), and their impact on firms' stock market values by Pakes (1985).

The structure of this paper is as follows. Section 1 describes the database that we have constructed and some of its key features. Section 2 sketches some simple models and the real options extensions that we use to estimate the effects of patenting on company performance. Section 3 details the econometric results and Section 4 gives some concluding comments. In short, we find considerable evidence of the importance of technology for firms' productivity and stock market performance. Higher uncertainty, as predicted, reduces this effect of patents on productivity but appears to have no significant effect on market value.

## 1. Data

We combine three principal datasets in constructing the IFS-Leverhulme database. Full details of the matching between the datasets is contained in Bloom and Van Reenen (2001), but we sketch the process here. The first dataset is the Case Western Patent data (see Trajtenberg *et al.*, 2000), the second is the Datastream

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[1] There is a small literature emerging on the impact of patents on company performance, such as the work on United Kingdom firms by Blundell *et al.* (1995), Bosworth, Greenhalgh and Stoneman Wharton (2000) and Bosworth *et al.* (2001) and on US firms by Hall *et al.* (2000).

[2] Hall *et al.* (1986) and Blundell *et al.* (1998) both provide evidence that patents are often applied for early on in the R&D process, so that further R&D expenditure may be needed to bring the products to market.

[3] See, in particular, Dixit and Pindyck (1994), Eberly and Van Mieghern (1997), and Bloom *et al.* (2000).

Table 1

*The Distribution of Firms by Total Patents,* 1968–1996

|  | 1 or more | 10 or more | 25 or more | 100 or more | 250 or more | 1,000 or more |
|---|---|---|---|---|---|---|
| Firms | 236 | 161 | 117 | 75 | 41 | 12 |

annual company accounting data, and the third is the Datastream daily share returns data.

To construct the patents database we used the computerised records of patents granted in the United States between 1968 and 1996. This is the largest electronic patent dataset in the world (the European Patent Office records begin only in 1976, and the records are patchy until the mid 1980s).

The second and third datasets contain the accounts and share returns of firms listed on the London Stock Exchange. From the population of public firms we selected those whose names began with the letters 'A' to 'L', which represents a random sample from the whole population. We also added in the top 100 R&D performing firms in the United Kingdom that were not already included in this list to maximise the numbers of patents we could collect. Ideally we would have collected information on all firms on the Stock Market, but the resource cost was too great. For all of these 415 firms we used 'Who Owns Whom' from 1985 to find the names of all subsidiaries.[4] We then used these subsidiary names to match to the Case Western Dataset by name.

### 1.1. *Patents and Citation Data*

The intersection of the two datasets gave us 236 firms who had taken out at least one patent between 1968 and 1996. The total number of patents taken out by this group over the entire period was 59,919, representing about 1% of the 6 million patents ever taken out at the US Patent Office. Table 1 shows that most of our group of United Kingdom firms are involved in a modest amount of patenting with about half the sample receiving more than 25 patents, while 12 firms received over 1,000 patents during the period. This concentration of innovative activity within large firms (the 12 account for 72% of all patents in our data), reflects a similar phenomenon in R&D expenditure where the 12 largest enterprises account for about 80% of all R&D expenditure.

The patents are graphed by their year of application in Fig. 1. The lesser degree of patenting activity in the latter part of the period reflects truncation

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[4] There are many problems with only using one year of data to match in the corporate structure. The process of matching is, however, extremely labour-intensive so it was only practical to perform it for one year. In future work we intend to also do the matching for later and earlier years.

Fig. 1. *Patents Per Year*

bias (on the right) because we collect statistics on patents granted. Since there is a delay between applying for and granting a patent of about two years, this leads to a downward bias towards the end of the period. There is also a truncation on the left of the graph as there may have been patents granted post 1968 which were applied for pre-1968. These caveats apart, there is little discernible trend in the total patents numbers granted to United Kingdom firms.

We also have data on the citations made by any of the other 6 million patents in the main dataset to our sample of 59,919 patents. Citations can be taken as an indicator of the technological value of a patent in that those patents which are frequently cited are likely to be more innovative and technologically productive. In Fig. 2 we plot the histogram of the lag between a patent being taken out and the subsequent citations to that patent. It can be seen that citations tend to happen relatively early on in a patent's life when the patent is widely known but technologically still innovative. Interestingly this citation lag still has not completely tailed off even after 20 years.

The five most cited patents are tabulated in Table 2 below with their patenting topic, the year they were granted and the number of cites made to them over the period 1976 until 1996.

The total number of citations to our patents, dated by the application year of the patent being cited, is plotted in Fig. 3. Because data on citations are only collected for patents granted after 1976 there is an early downward bias reflecting the fact
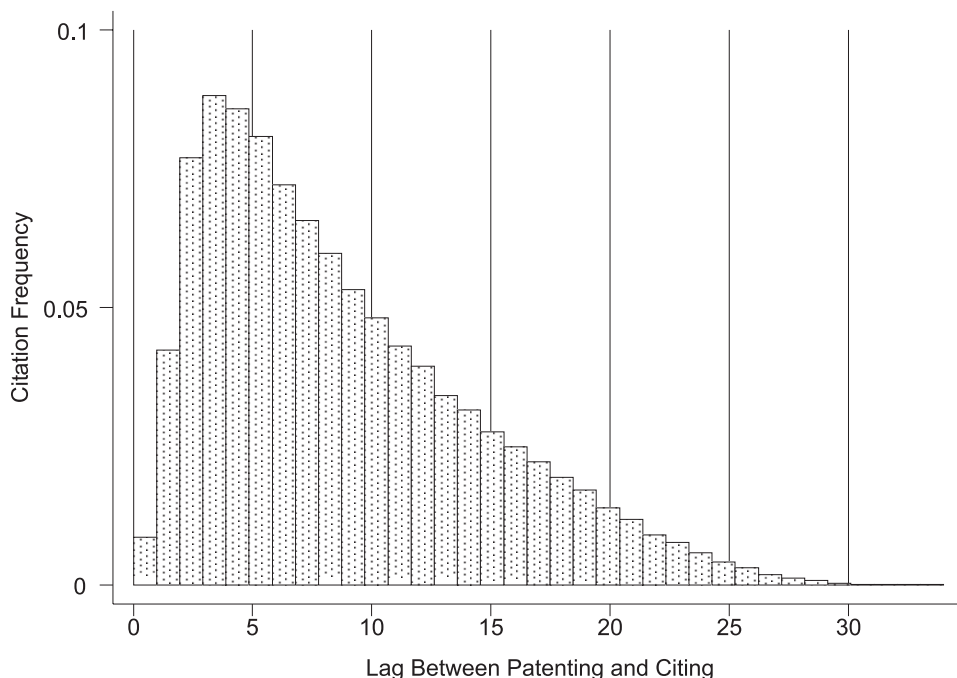
Fig. 2. *Lags Between Patenting and Citing*

Table 2
*The Top Five Cited Patents*

| Company | Patent Topic | Grant Year | Cites 1976–96 |
|---|---|---|---|
| Shell | Synthetic resins | 1972 | 221 |
| Grand Metropolitan | Microwave heating package | 1980 | 174 |
| ICI | Herbicide compositions | 1977 | 130 |
| Unilever | Anticalculus composition | 1977 | 97 |
| British Oxygen Corp. | Pharmaceutical drugs | 1975 | 89 |

that for patents granted pre 1976 some of the initial citations data are missing. The discussion in the paragraph above and Fig. 2 suggests that the loss of these early citations could lead to a serious downward bias for patents taken out pre 1976 since for them this would represent a period of relatively high citation activity. There is also a tail end bias as patents applied for towards the end of the period will only be part of the way through their citations lifecycle, and so will have been cited less often by 1996.

To deal with these biases we use a non-parametric series estimator based on a full Fourier sine and cosine expansion. We assume that the total lifetime number of citations per patent per year is constant throughout our sample. Therefore any observed changed in the observed aggregate citation levels will be due to time
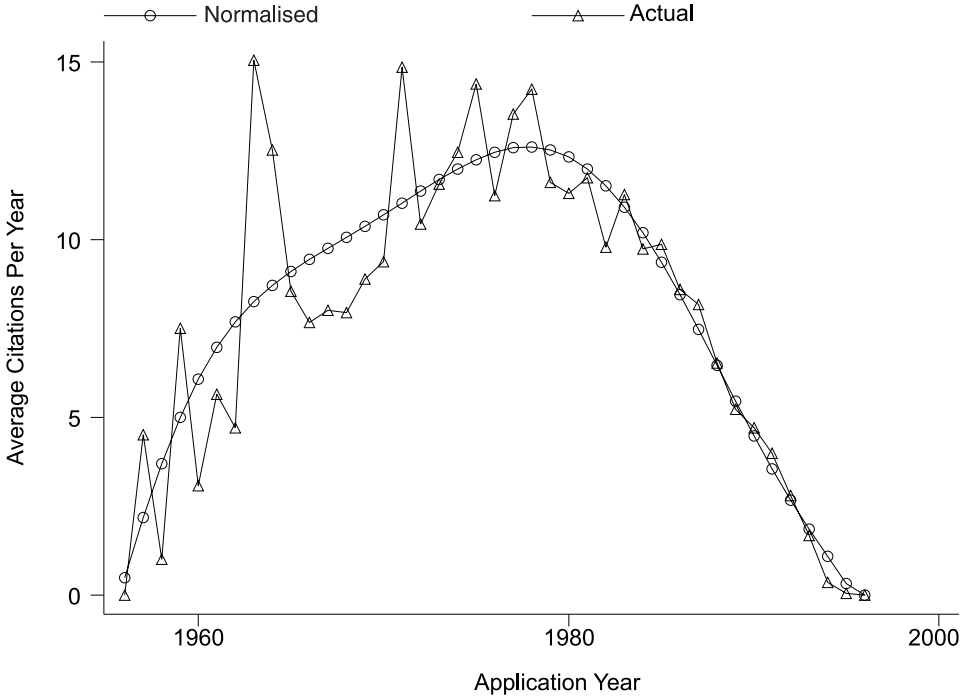
Fig. 3. *Actual and Normalised Citations per Year*

varying levels of truncation bias. We also assume that this time varying truncation bias varies smoothly over time according to some piecewise continuous function of time.[5] Our normalisation estimator then uses a Fourier expansion to fit a smooth curve to the observed time variation in aggregate citation levels to non-parametrically estimate a truncation bias function.

A Fourier expansion was used because of its ability to approximate conveniently to an arbitrary degree of accuracy any piecewise-continuous function (see Churchill and Brown, 1987). The actual number of yearly citations was regressed against these eight Fourier series terms and the predicted value taken as our normalising function.[6] The smoothing property of our estimator can be seen in Fig. 3 which plots the actual citation frequency and our non-parametric functional estimator. This functional estimator of the time varying citation bias is then inverted to

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[5] That our *observed* citation frequency is not smooth over time, even in our sample of almost 60,000 patents, is testament to the extreme skew of the citations data. In datasets such as these which have large second moments the usual weak convergence of the empirical distributions to their underlying distribution is extremely slow (see for example Billingsley, 1986), so that smoothing is usually required.

[6] Increasing the length of the base period or using the first three or five terms does not have any significant impact on our results. This is because the first few terms of the Fourier expansion drive the results, as noted for example by Bertola and Caballero (1994) in a related application. The procedure itself is very straightforward, just requiring an OLS regression of the yearly citation frequency against the Fourier terms: Cos(base), Sin(base), Cos(2×base), Sin(2×base) etc... Further details on implementation can be found in Kreyszig (1999).

Table 3

*Correlations Between Knowledge Stock Measures Normalised by Firm Employment*

|  | Pat Stock | Cite Stock | 5 Year Cite Stock | R&D |
|---|---|---|---|---|
| Patent Stock | 1 | 0.9688 | 0.9074 | 0.5001 |
| Cite Stock |  | 1 | 0.9453 | 0.4829 |
| 5 Year Cite Stock |  |  | 1 | 0.4032 |
| R&D flow |  |  |  | 1 |

*Notes*: All knowledge variables have been size normalised by total firm employment. Non-normalised correlations are all higher. R&D correlations are only for those years in which R&D is reported.

re-weight the citations per patent. This ensures that the normalised citations per patents remain approximately constant over the period.

In calculating a patent based proxy for knowledge stocks it is also more sensible to use a stock measure rather than a flow measure of knowledge as the benefits from a patent are likely to persist into future years. We calculate a set of preferred measures of the stock of patents through the perpetual inventory method with a knowledge depreciation rate, $\delta$, set to 30% as in, for example, Cockburn and Griliches (1988). The same perpetual inventory method is used to calculate the citation stock where the flow variable is the citation weighted number of patents. The '5 year cite stock' uses only the first 5 years of citations (after an application) to obtain a citation weighting but without any normalisation. Since we select our citation estimation period to run up to 1990 only whilst our citing data run up to 1996 this means we have 5 years of observations on citations for every patent so that no truncation bias correction will be needed for this 5 year measure.

It is comforting that our three measures of the knowledge stock – the patent stock, the citation weighted patent stock, and the 5 year citation weighted stock – have a strong correlation as demonstrated in Table 3. They also have a strong correlation with R&D expenditure. This suggests that whilst each should have its own merit in capturing various aspects of the knowledge stock they proxy a similar measure of the technological innovation stock.

1.2. *Firm Level Accounting and Uncertainty Data*

The company data are drawn from the Datastream on-line service and represent the accounts of firms listed on the United Kingdom stock market. Our initial sample of 415 firms (those whose names began with A–L or were large R&D performers)[7] for which we matched patent data was then cleaned, leaving a sample of 404 firms, to which 184 were matched as having patenting subsidiaries (see Bloom and Van Reenen (2001) for details of this cleaning and matching process).[8]

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[7] See Bloom and van Reenen (2001) for details of this sample selection, cleaning and matching process.

[8] This is less than our group of 236 patenters because of both the loss of some firms due to trimming and the loss of some years of observations for the remaining firms due to the unavailability or poor quality of data on employment in the early 1970s.

Table 4

*Descriptive Statistics for the 184 Patenting Firms, 1969–1996*

|  | median | mean | stan. dev. | min. | max |
|---|---|---|---|---|---|
| Real Capital (£m) | 144 | 745 | 1,778 | 1.6 | 18,514 |
| Employment (1000s) | 8,279 | 23,963 | 41,566 | 40 | 312,000 |
| Real Sales (£m) | 362 | 1,224 | 2,494 | 1.15 | 20,980 |
| Real Market Value (£m) | 153 | 740 | 1,766 | 0.29 | 19,468 |
| Patents | 3 | 12.6 | 34 | 0 | 409 |
| Total Citations | 13.7 | 61.2 | 157 | 0 | 1,808 |
| Patent Stock | 10 | 42.6 | 113 | 0 | 1,218 |
| Cite Stock | 49.2 | 202 | 507 | 0 | 5,157 |
| 5 Year Cite Stock | 26.2 | 105.9 | 227 | 0 | 2,919 |
| Uncertainty | 1.39 | 1.47 | 0.42 | 0.60 | 6.6 |
| Observations per firm | 22 | 20 | 7.6 | 3 | 29 |

*Notes*: Capital, sales and market value are all in 1985 million. Patents is the total number of patents per firm year whilst cites is the normalised total number of citations to a firm's patents per year. Uncertainty is the % standard deviation of daily share returns. Sample covers years 1968–96.

Table 4 reports summary statistics for this set of 184 patenting firms. From the last row of the table it can be seen that we generally have a long time series of data on each firm – on average over 20 years for each firm. The patent numbers demonstrate the large variation in patenting per firm year with some firms only taking out sporadic patents – as demonstrated by the zero patent observations – and others taking out 409 patents in a single year (ICI in 1974). The total cites number represents the normalised sum of citations for all patents taken out in each firm year.

In measuring uncertainty we have to measure firms' uncertainty about future prices, wages rates, exchange rates, technologies, consumer tastes and government policies. In an attempt to capture all factors in one scalar proxy for firm level uncertainty we use the variance of the firm's daily stock returns,[9] denoted $\sigma_i^2$. In accordance with the standard assumptions of theories of real options this is a time invariant but firm specific proxy for uncertainty. This measure includes on a daily returns basis the capital gain on the stock, dividend payments, rights issues, and stock dilutions. Such a returns measure provides a forward looking proxy for the volatility of the firm's environment which is implicitly weighted in accordance with the impact of these variables on profits. A stock returns-based measure of uncertainty is also advantageous because the data are accurately reported at a sufficiently high frequency to provide an extremely accurate measure. Our sampling size of 265 recordings per year for the 22 year life of our average firm therefore provides an extremely low sampling variance.[10]

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[9] This measure of uncertainty is also used by other papers in the literature on uncertainty and investment, such as Leahy and Whited (1998).

[10] For example, Andersen and Bollerslev (1998) use high frequency exchange rate data with 288 recordings per period and calculate that the implied measurement errors are less than 2.5% of the true volatility.

## 2. Models of Patents and Company Performance

We work with a simple Cobb–Douglas production function of the form

$$Q = AG^\alpha N^\beta K^\gamma \tag{1}$$

where $Q$ is real sales, $G$ is the knowledge stock, $N$ is number of employees, $K$ is the capital stock and $A$ is an efficiency parameter. Taking logs and introducing subscripts for firm $i$ at time $t$ we have

$$\log Q_{it} = \log A_{it} + \alpha \log G_{it} + \beta \log N_{it} + \gamma \log K_{it}. \tag{2}$$

We parameterise efficiency, $A_{it} = \exp(\eta_i + \tau_t + v_{it})$, as a function of firm specific fixed effects ($\eta_i$), time effects ($\tau_t$) and a random stochastic term ($v_{it}$). In our empirical application we use patent stocks and citation-weighted patent stocks ($PAT$) as empirical proxies of $G$, the knowledge stock.

$$\log Q_{it} = \alpha \log PAT_{it} + \beta \log N_{it} + \gamma \log K_{it} + \eta_i + \tau_t + v_{it}. \tag{3}$$

We estimate (3) by within groups (least squares dummy variables) correcting the standard errors for heteroscedasticity.

Market value equations are less well established than production functions. The standard approach pioneered by Griliches (1981) is based on a specification of the form (see also Hall *et al.*, 2000; Bosworth, Greenhalgh and Wharton, 2000)

$$\log\left(\frac{V}{K}\right)_{it} = \delta\left(\frac{G}{K}\right)_{it} + \eta_i + \tau_t + v_{it} \tag{4}$$

where $V$ is the market value of the firm. The left hand side of (4) is essentially Tobin's average Q. Hi-tech firms with high levels of intangible knowledge capital will have a larger market value than one would expect from their fixed capital stocks.

### 2.1. *Uncertainty and Real Options*

The two models laid out above assume that the knowledge contained in patents can be immediately used and acted on by firms. Patents, however, represent new products or process innovations whose introduction can involve sizeable investments in additional plant and equipment, hiring and retraining workers, and advertising and marketing. Much of this expenditure will be irreversible – once it is undertaken the initial costs will not be recoverable. Thus, when firms are facing uncertain market conditions then they will possess patent *real options*.[11] These patent *real options* reflect the value a firm places on its ability to choose the timing of its investment in its patented technologies when this involves sunk costs.

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[11] Apart from partial irreversibility and market uncertainty, the third condition for the existence of real options – that firms can delay their actions – is clearly satisfied in this case where patents give firms the exclusive rights to use their innovations until their patents expire.

A large theoretical literature has grown from the seminal papers of McDonald and Siegel (1986), Bertola (1988), Pindyck (1988), and Dixit (1989) demonstrating the important role such real options can play in firm's optimal investment strategies. As a result real options should also play an important role in our approach to modelling investment in innovation. This work emphasises the role real options play in retarding the response of firms to changing market conditions.[12] When market conditions are uncertain firms become reluctant to commit large sums to new investment projects or dismantle old investment projects in case conditions change. This leads to 'cautionary' investment behaviour. This 'cautionary' effect of real options in retarding the response to changing market conditions has been confirmed empirically for physical investment by Guiso and Parigi (1999) in a cross section of Italian firms, and by Bloom *et al.* (2001) in a panel of UK firms.

To incorporate these real options effects we extend the concept of knowledge stock into embodied knowledge and disembodied knowledge. Embodied knowledge represents those product and process innovations which the firm has invested in. Disembodied knowledge, however, represents the remaining ideas which the firm has under patent but has not yet committed into actual production. When conditions are highly uncertain the firm will be more cautious because of the value of the real options associated with embedding new innovations into production. We develop a stylised model that illustrates the impact of patenting real options on market values, production and embodiment. This model has been kept deliberately simple to ensure a closed form analytical solution, but could potentially be extended in a number of directions.

The firm's value is assumed to depend on its collection of embodied patents, $P_k$, $k = 1 \dots K$, and disembodied patents $P_j$, $j = 1 \dots M$, where $P_k$ is the profit flow from patent $k$ if embodied. Disembodied patents are those that the firm owns the intellectual property rights to, but would need a sunk cost development of $I$ to start producing their potential profit flows of $P$. Embodied patents have already been developed and produce a continuous flow of profits $P$ for the firm. Thus, the firm's value (*VAL*) can be written as[13]

$$VAL(P_1, P_2, ..., P_{K+M}) = \sum_{k=1}^{K} V^E(P_k) + \sum_{j=1}^{M} V^D(P_j) \qquad (5)$$

where $V^E(.)$ and $V^D(.)$ are the values of embodied and disembodied patents.

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[12] See, for example, the work on threshold behaviour by Dixit and Pindyck (1994), and Bloom (2000), Abel and Eberly (1996).

[13] Of course the firm's value should also be a function of a number of other state variables such as its capital stock, employees, interest rates and other factor prices. However, in order to keep the patent real options analysis tractable we have ignored these factors, delivering stylised results. A more general approach which includes other factors is taken by Eberly and Van Mieghem (1997) and Bloom *et al.* (2001), which also predicts a similarly strong retardation effect of real options on firm responses.

New patents are assumed to arrive continuously in a stochastic manner. For simplicity these new patents (and their associated innovations) are assumed to arrive at an exogenous rate[14] with a *potential* embodied profit flow rate of $P$, which is initially drawn from a cumulative distribution $H(P)$. We assume that this initial distribution of new patents has a large enough support so that some new patents are so valuable that they are immediately embodied.

For each patent its (potential) embodied profit flow evolves stochastically as market conditions change, and is assumed to follow a geometric Brownian motion process

$$dP = \mu P dt + (\sigma dZ_k + \sigma_F dZ_F) \tag{6}$$

where $dZ_i$ and $dZ_F$ are independent patent and firm level Weiner processes.[15] These represent separate patent and firm level shocks – so that, for example, for a pharmaceutical firm patent level shocks would just affect the value of the particular drug while firm level shocks would affect the value of every drug in its portfolio. Since these two processes are assumed to be independent overall uncertainty can be written as $\sigma^2 = (\sigma_k^2 + \sigma_F^2)$. The value of patents that are already embodied can be calculated as $V^E(P) = P/(\rho - \mu)$ where $\rho$ is the firm's cost of capital and $\mu$ is the mean growth of patent profits.[16] To derive the value of disembodied patents, which can be thought of as a patent option, we derive the differential equation describing its value function $V^D(P)$, which comprises only an expected gain term since there is no profit flow for disembodied patents:

$$V^D(P) = e^{-\rho dt} \mathrm{E}[V^D(P + dp)] \tag{7}$$

$$= V^D(P) + \mu V_P^D(P) dt + \frac{\sigma^2}{2} V_{PP}^D(P) dt - \rho V^D(P) \quad \text{in } \lim dt \to 0.$$

The solution to this takes the form $V^D(P) = AP^\beta$, where $A$ is a constant, and $\beta > 1$ is the solution to the characteristic equation.[17]

Therefore, the firm value can also be defined to be

$$VAL(P_1, P_2, ..P_{K+M}) = \sum_{k=1}^{K} \frac{P_k}{\rho - \mu} + \sum_{j=1}^{M} AP_j^\beta. \tag{8}$$

Sales are assumed to be representable as a multiple of profits due to markup pricing, so that we can define some $\lambda$ so that

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[14] We could allow the arrival rate of new patents to be endogenously determined, for example by allowing firms to vary their R&D spend. However, this would introduce more state and control variables into the dynamic programme and preclude a straightforward analytical solution.

[15] Weiner processes are stochastic white noise processes. This specification allows for patent level and common firm level stochastic shocks. Independence between these two processes considerably simplifies the mathematics but is not essential for the results.

[16] In fact if patents are (more realistically) modelled as having a fixed expiring date of $T$ then their value would be $[P_k/(\rho - \mu)](1 - e^{\rho T})$. Since expiry does not change the qualitative implications of our results we ignore this for analyical simplicity.

[17] See McDonald and Siegel (1986) and Dixit and Pindyck (1994) for more details on solving these option value problems.

$$SALES = \lambda \sum_{k=1}^{K} P_k. \tag{9}$$

Solving the firm's dynamic programme we find that there will exist some value of embodied profit flow $P^*$ at which it will become optimal for the firm to pay the sunk embodiment cost $I$ and start receiving this profit flow. To solve for this embodiment value $P^*$ we need to derive two optimality conditions. The first is the value matching condition which requires that at $P^*$ the option value just equals the discounted profit flows less the sunk cost of embodiment,

$$AP^{*\beta} = \frac{P^*}{\rho - \mu} - I. \tag{10}$$

The second condition, known as the smooth pasting condition, takes another derivative, and ensures embodiment is optimally timed,

$$\beta AP^{*\beta-1} = \frac{1}{\rho - \mu}. \tag{11}$$

Combining these two conditions allows us to solve for this optimal embodiment profit flow $P^*$

$$P^* = \frac{\beta}{\beta - 1} I(\rho - \mu). \tag{12}$$

This demonstrates the option value effect whereby investment in the patent will only occur after the embodied profit flow has risen to $\beta/(\beta - 1)$ times $I(\rho - \mu)$, compared to the no real option case in which embodiment would occur when $P^* = I(\rho - \mu)$, which is the flow cost of embodiment. This option value multiple $\beta/(\beta - 1)$ is increasing in $\sigma^2$ so that the embodiment threshold is higher in more uncertain environments.[18]

Using this model we can predict the signs of the empirical relationships between sales, market values, patenting, and uncertainty. First, the firm's valuation is clearly increasing in patent numbers, $PAT$ (which equals $K + M$ in the model above), since even disembodied patents have an option value. Since market values are forward looking this effect will take place immediately, so that integrating with respect to the initial patent valuation we can say the impact patenting effect on market values will be positive,

$$\frac{\partial VAL}{\partial PAT} = \int_0^{P^*} V^D(P) dH(P) + \int_{P^*}^{\infty} V^E(P) dH(P) > 0. \tag{13}$$

Firms' sales will also be increasing, in expectation, in the number of patents since some new patents will have a sufficiently high initial value that will be embodied immediately. Given the embodiment threshold $P^*$ and assumption on the distribution of initial values $H(x)$, the impact value of new patents on sales will be

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[18] See McDonald and Siegel (1986) or Dixit and Pindyck (1994).

$$\frac{\partial SALES}{\partial PAT} = 1 - H(P^*) > 0. \tag{14}$$

The first order derivative of market value with respect to uncertainty will also be positive since higher uncertainty will increase the option value of disembodied patents[19]

$$\frac{\partial VAL}{\partial \sigma^2} = \sum_{i=1}^{N} \frac{\partial V(P_i)}{\partial \sigma^2} > 0. \tag{15}$$

The first order derivative of sales with respect to uncertainty depends on the extent to which additional patents are embodied. These effects can be ambiguous. On the one hand higher uncertainty increases the embodiment threshold $P^*$, which will directly reduce the rate of patent embodiment. On the other hand, higher uncertainty will make the potential embodied profit flows $P$ more volatile, and so increase the chance that any patent actually hits its embodiment threshold. Overall, as Abel and Eberly (1999) show in a related physical investment model, these effects can go in either direction:

$$\frac{\partial SALES}{\partial \sigma^2} \lessgtr 0. \tag{16}$$

Finally we are also interested in the cross derivative of new patenting and uncertainty. The cross derivative for market value is again positive since higher uncertainty will increase the value of extra patents. This impact will be felt immediately since market values are forward looking, so that

$$\frac{\partial^2 VAL}{\partial PAT \partial \sigma^2} > 0. \tag{17}$$

The cross derivative of sales with respect to patenting and uncertainty will be negative because of the real options effect on embodiment. Higher uncertainty will raise the embodiment threshold $P^*$ which will reduce the fraction of new patents that are immediately embodied. Taking the first derivative of equation (14) with respect to uncertainty we can show this will be negative

$$\frac{\partial^2 SALES}{\partial \sigma^2 \partial PAT} = -h(P^*) \frac{dP^*}{d\sigma^2} < 0 \tag{18}$$

where $h(P^*)$ is the probability distribution derived from $H(P)$.

This stylised model focuses only on patents as a driver for productivity, but our empirical specification allows, of course, for an independent role for the other factors of production. We assume that the augmented Cobb–Douglas production function can take the form:

$$\log Q_{it} = \alpha \log PAT_{it} + \beta \log N_{it} + \gamma \log K_{it} + \psi \sigma_i + \chi(\sigma_i * \log PAT_{it}) + \eta_i + \tau_t + \upsilon_{it} \tag{19}$$

where the coefficients $\psi$ and $\chi$ will pick up the direct and interaction effects of uncertainty. The coefficient $\psi$ is theoretically ambiguous in sign while the

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[19] Of course in a more general model we could allow the firm's discount rate to vary with the level of uncertainty, which could lead higher uncertainty to reduce market values if most patents were already embodied.

Table 5

*Basic Production Functions*

| Log Real Sales | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
|---|---|---|---|---|---|---|---|
| Firms | All | All | | | Patenters Only | | |
| Log Capital | 0.330*** | 0.287*** | 0.433*** | 0.435*** | 0.463*** | 0.467*** | 0.463*** |
| | (0.006) | (0.010) | (0.027) | (0.027) | (0.031) | (0.031) | (0.031) |
| Log Employment | 0.649*** | 0.605*** | 0.550*** | 0.546*** | 0.496*** | 0.493*** | 0.496*** |
| | (0.006) | (0.010) | (0.027) | (0.027) | (0.031) | (0.031) | (0.031) |
| Log Patent stock | | | | 0.027*** | | | −0.009 |
| | | | | (0.008) | | | (0.019) |
| Log Cite stock | | | | | 0.031*** | | 0.038** |
| | | | | | (0.008) | | (0.016) |
| Log 5 Year Cite Stock | | | | | | 0.032*** | |
| | | | | | | (0.007) | |
| Firm dummies | no | yes | yes | yes | yes | yes | yes |
| Time dummies | yes | yes | yes | yes | yes | yes | yes |
| Adj. R-Squared | 0.901 | 0.989 | 0.992 | 0.992 | 0.992 | 0.992 | 0.992 |
| No. observations | 18,007 | 18,007 | 2,222 | 2,222 | 1,900 | 1,900 | 1,900 |
| No. firms | 1,725 | 1,725 | 178 | 178 | 165 | 165 | 165 |

*Notes*: The dependent variable is 'log real sales'. Columns (1) and (2) present results using our complete Datastream population of all firms, Columns (3) to (7) present the results for our sub-sample of firms with patents. The estimation period covers 1968 until 1993 inclusive for columns (1) to (4), and 1968 until 1990 inclusive for columns (4) to (7) which use the citation data. The symbol *** denotes 1% significance, ** denotes 5% significance and * denotes 10% significance. Standard errors are robust to arbitrary heteroscedasticity.

interaction coefficient $\chi$ is predicted to be negative. Note that we will not be able to identify the linear effect of uncertainty from $\eta_i$ separately in the specifications where the latter are treated as fixed effects.

In our empirical market value equation shown below, which includes uncertainty interactions, real options theory predicts positive coefficients $\theta$ and $\zeta$ on linear uncertainty and the interaction term.

$$\log\left(\frac{V}{K}\right)_{it} = \delta\left(\frac{G}{K}\right)_{it} + \theta\sigma_i + \zeta\left[\sigma_i * \log\left(\frac{G}{K}\right)_{it}\right] + \tilde{\eta}_i + \tilde{\tau}_t + \tilde{\upsilon}_{it}. \tag{20}$$

## 3. Results

Table 5 presents the results of estimating a standard production function on our sample of firms. Column (1) has the OLS estimates of the production function for our complete population of over 2,000 Datastream firms. As expected the coefficients on capital and labour are both positive and significant at conventional levels, and their sum is close to unity (suggesting constant returns in tangible factors). In column (2) we undertake estimation with our preferred within groups estimator which controls for time invariant differences between firms by including firm dummies. Again the coefficients on capital and labour are positive and significant, although slightly smaller than in column (1). Column (3) compares these within groups results from the whole Datastream sample to our sub-sample of patenters. The higher point estimates on capital and lower point estimates on labour imply

Table 6

*Market Value with Patents Measures*

| $log(V_{i,t}/K_{i,t-1})$ | (1) | (2) | (3) | (4) | (5) |
|---|---|---|---|---|---|
| Patent Stock/Capital | 1.221** (0.492) | | | −0.533 (0.755) | 1.002** (0.492) |
| Cite Stock/Capital | | 0.345*** (0.140) | | | |
| 5 Year Cite Stock/Capital | | | 0.435** (0.206) | 0.443** (0.228) | |
| Cite Stock/Patent Stock (ave. cite per patent) | | | | | 0.015 (0.010) |
| Firm dummies | yes | yes | yes | yes | yes |
| Time dummies | yes | yes | yes | yes | yes |
| No. observations | 2,138 | 1,821 | 1,821 | 1,821 | 1,821 |
| No. firms | 172 | 158 | 158 | 158 | 158 |

*Notes*: The dependent variable is 'log (market value/lagged capital)'. Due to the need for a lagged capital observation the estimation period covers 1969 until 1994 inclusive for column (1), and 1969 until 1990 inclusive for columns (2) to (4) (which use the citation data which are only available for this shorter period). The symbol *** denotes 1% significance, ** denotes 5% significance and * denotes 10% significance. Standard errors are corrected for arbitrary heteroscedasticity.

that our sample of patenting firms are on average more capital intensive than lower tech firms (as one would expect). In fact our patenting firms have on average a 20% higher capital to labour ratio than non patenting firms.

The last four columns of Table 5 report the results from including patents as a proxy for knowledge in the production function. In column (4) we use patent stocks, in column (5) citation weighted patent stocks and in column (6) the five year ahead citation weighted patent stock measure.[20] On all the alternative measures, patent stocks are significant at the 0.05 level with an elasticity of about 0.03. This suggests that a doubling of the patents stock would lead to a 3% increase in total factor productivity.[21] In column (7) we include both the patent stock and the citation knowledge stock and find that patents are no longer significant. Thus, citations provide significant information over and above raw patents numbers. This suggests citations could provide a valuable proxy for evaluating knowledge stocks and tracing knowledge flows.

Table 6 reports the results of estimating the impact of patents on market values using the conventional average Q specification described in (4). In column (1) we use the patent stock measure, in column (2) our citation weighted patents stock measure, and in column (3) the five year ahead measure and find all three have significant explanatory power at the 5% level. The coefficient in column (2) suggests, for example, that doubling the citation weighted patents stock would increase the value of firms per unit of capital by about 35%. This large estimate of

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[20] We also re-estimate this equation using the wage bill rather than employment as the labour variable to reflect potential skills differentials across employers. This yields extremely similar results with a slightly higher and still 1% significant patent citation point estimate of 0.0329 (0.0076) for the specification in column (5).

[21] We are grateful to a referee for pointing out that what we in fact measure is *revenue productivity* which includes both changes in factor productivity as well as any increase in the markup firms are able to charge consumers from new innovations.

the effect of cited patents on market values captures the market's expectation of the total discounted rents from patented innovations.[22] In column (4) we again compare the predictive power of patents and citation weighted patents and find that citations provide significant additional information over and above raw patents counts. Column (5) follows Hall *et al.* (2000) by decomposing the citation weighted patent stock into a patent stock measure and an average cites per patent stock measure. It can be seen that the raw patent count provides the bulk of the information with the average cite per patent measure positive but significant only at the 15% level.

In Table 7 we conduct some robustness tests on our basic models. In columns (1) and (2) we include both the patent stock and the lagged patent stocks measures. It is the lagged variable which is most informative in predicting productivity, suggesting that patented innovations take some time to enter the production function. In the market value equation, however, the current value of patents per unit capital has the larger coefficient (1.173) and is significant at the 15% level, while the lagged value has a coefficient of 0.656 and is not significant at all.[23] This larger point estimate on the current value in the market value equation appears to reflect the forward looking nature of the market value measure. In columns (3) and (4) we lag all our right hand side variables one period to control for the possible endogeneity of current values of the explanatory variables. This does not noticeably change our results with significant effects of patents on productivity and market values. We also re-run this specification with all our explanatory variables lagged twice and again find our results look very similar with a point estimate (standard error) of 0.042 (0.013) on patents in the productivity equation and of 1.01 (0.405) on (patents/capital) in the market value equation.[24] We also look for both structural breaks and a time varying coefficient on our patent measures, and somewhat surprisingly, find no significant evidence for either.

Finally, Table 8 reports our results from investigating the effects of uncertainty on the productivity response to patenting. In column (1) the patenting uncertainty interaction term takes the predicted negative sign in our productivity equation, and is significant at the 5% level. The coefficient on the level of un-

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[22] These results are larger than those reported for US firms by Hall *et al.* (2000) where they report coefficients of 0.607 and 0.108 on (patent/capital stock) and (cite patent/capital stock). One reason could be that the Hall *et al.* (2000) study uses a different sample with a higher weighting in smaller NASDAQ firms. Another reason appears to be because they chose a 15% rather than a 30% depreciation rate on patents so that their patenting and citation stocks will be approximately twice our size. Our results are robust to using this alternative assumption on the knowledge depreciation rate. For example, if we use a 15% rather than a 30% depreciation rate and re-estimate our market value equations we obtain a coefficient (standard error) of 0.879 (0.327) and 0.246 (0.081) on the (patent stock/capital stock) and (citation stock/capital stock) terms respectively. In our productivity equations we obtain a coefficient (standard error) of 0.035 (0.013) and 0.028 (0.010) on our patent stock and citation stock measures respectively.

[23] Although individually insignificant, the current and lagged values are jointly significant at the 1% level.

[24] Because of the limited cross-sectional size of our dataset the standard GMM estimators would not be appropriate. However, we did undertake some exploratory IV estimations using orthogonal deviations which yielded approximately similar point estimates but with much larger standard errors (see Bloom and Van Reenen (2001) for details).

Table 7

*Robustness Checks*

|  | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
|  | *Real Sales* | $log(V_{i,t}/K_{i,t-1})$ | *Real Sales* | $log(V_{i,t}/K_{i,t-1})$ |
| Log Real Capital | 0.463*** | | | |
|  | (0.028) | | | |
| *Lagged* Log Real Capital | | | 0.439*** | |
|  | | | (0.031) | |
| Log Employment | 0.531*** | | | |
|  | (0.028) | | | |
| *Lagged* Log Employment | | | 0.447*** | |
|  | | | (0.030) | |
| Log Patent Stock | −0.006 | | | |
|  | (0.011) | | | |
| *Lagged* log Patent Stock | 0.030** | | 0.040*** | |
|  | (0.010) | | (0.010) | |
| Patent Stock/Capital | | 1.173 | | |
|  | | (0.787) | | |
| *Lagged* Patent Stock/Capital | | 0.656 | | 1.369*** |
|  | | (0.721) | | (0.529) |
| *Firm* dummies | yes | yes | yes | yes |
| Time dummies | yes | yes | yes | yes |
| No. observations | 2,055 | 1,975 | 2,055 | 1,975 |
| No. firms | 175 | 169 | 170 | 169 |

*Notes*: The dependent variable for columns (1) and (3) is 'log real sales' and the dependent variable for columns (2) and (4) is 'log (market value/capital stock)' – both are in 1985 prices. The estimation period covers 1968 until 1990 inclusive for columns (1) and (3), and 1969 until 1990 inclusive for (2) and (4). The symbol *** denotes 1% significance, ** denotes 5% significance and * denotes 10% significance. Standard errors are corrected for arbitrary heteroscedasticity.

certainty ($\sigma_i$), which is theoretically ambiguous, is also negative but not significant at the 5% level. When we move to the within groups specification in column (2) by including a full set of firm dummies we have to drop this firm specific uncertainty term $\sigma_i$ since this is collinear with the firm dummies. In column (2) we see that the patenting interaction term is, as before, negative and significant at the 1% level. The size of this interaction coefficient (−0.01) suggests that increasing a firm's uncertainty by one standard deviation (0.42) from the median level of uncertainty (1.39) would reduce the elasticity of productivity with respect to patents from 0.024 to 0.020. Hence, for a one standard deviation increase in uncertainty the patenting effect on productivity falls by about 20%, a moderate but not enormous change.

In column (3) we investigate the levels and interaction effects of uncertainty on the market value in an OLS equation. We find that both the direct effect and the interaction effect have a negative impact on market values. This is in contradiction to our theory which predicts a positive relationship, suggesting that uncertainty may play a more powerful negative role through some other channel such as the cost of capital. The study by Griliches *et al.* (1988) finds, in fact, that while patents are significant in determining market values they account for only about 5% of their variance. In contrast variation in the cost of capital will probably account for a

Table 8

*Real Options Effects of Uncertainty*

|  | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
|  | Real Sales | Real Sales | $log(V_{i,t}/K_{i,t-1})$ | $log(V_{i,t}/K_{i,t-1})$ |
| Log Real Capital | 0.457*** | 0.451*** |  |  |
|  | (0.015) | (0.021) |  |  |
| Log Employment | 0.511*** | 0.541*** |  |  |
|  | (0.017) | (0.017) |  |  |
| Log Patent Stock | 0.024** | 0.040*** |  |  |
|  | (0.011) | (0.008) |  |  |
| $\sigma_i \times$ Log Patent Stock | −0.015** | −0.010*** |  |  |
|  | (0.006) | (0.003) |  |  |
| Patent Stock/Capital |  |  | 0.930*** | 1.436*** |
|  |  |  | (0.331) | (0.417) |
| $\sigma_i \times$ Patent Stock/Capital |  |  | −0.279* | −0.123 |
|  |  |  | (0.156) | (0.126) |
| $\sigma_i$ | −0.036 |  | −0.319*** |  |
|  | (0.024) |  | (0.047) |  |
| Firm dummies | no | yes | no | yes |
| Time dummies | yes | yes | yes | yes |
| No. observations | 2,121 | 2,121 | 2,121 | 2,121 |
| No. firms | 171 | 171 | 171 | 171 |

*Notes*: The dependent variable for the first two columns is 'log real sales' and the dependent variable for the second two columns is 'log real market value' – both are in 1985 prices. The estimation period covers 1970 until 1993 inclusive. The symbol *** denotes 1% significance, ** denotes 5% significance and * denotes 10% significance. Standard errors corrected for arbitrary heteroscedasticity.

much larger share of the variation in market value.[25] In column (4) we include a full set of firm dummies to control for fixed differences between firms, and the uncertainty–patenting interaction term remains negative but is insignificant at conventional levels.

To account for the possible effects of market-wide bubbles and fads we also calculate a second measure of uncertainty, using the variance of the firm's daily share returns normalised by the return on the FTSE All-Share index. This measure eliminates common stock market volatility. Results using this normalised measure are almost identical to those reported in Table 8, and are available on request from the authors.

## 4. Conclusions

Patents citations are a potentially powerful indicator of technological innovation. Our analysis of the new IFS-Leverhulme database on over 200 major British firms since 1968 has uncovered some interesting results. First, we show that patents have had an economically and statistically significant impact on firm-level productivity

[25] Strictly speaking the relationship between uncertainty and capital valuation implied by theories such as the Capital Asset Pricing Model (CAPM) or the Consumption CAPM relies on *covariance* (with the market) rather than *variance*. Since covariances and variances are likely to be positively linked, however, this negative statistical relationship between variance and capital valuation is not surprising.

and market value. For example, a doubling of the citation-weighted patent stock increases total factor productivity by 3%. We find that citations are more informative than the simple patent counts that have been used previously in the literature. Secondly, we find that while patenting feeds into market values immediately it appears to have a slower effect on productivity. Thirdly, we find that higher market uncertainty reduces the impact of new patents on productivity. This is consistent with a simple 'real options' effect that has been found to be important in the literature on tangible investment.

There are several future directions to take this stream of research. We have not investigated the technological spillovers that have been a focus of attention in the recent literature. Patent citations are a potentially useful source of information in tracking the flows of knowledge across industries and countries and we intend to use the citations data in combination with R&D to investigate spillovers. A second area of interest is in probing the uncertainty results in more detail. If more uncertain environments reduce the productivity benefits from patents then it is likely that reductions in uncertainty will imply a larger effect on firms' incentives to innovate. A natural extension of this work is to augment the patenting equations with measures of uncertainty to uncover the importance of volatility in affecting innovation. Finally, the results presented here imply that the Government's attempts to reduce uncertainty (if they work in lowering 'boom and bust') will have a direct effect on productivity through increasing the productivity impact of Britain's knowledge capital as measured by citation weighted patents.

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# References

Abel, A. and Eberly, J. (1996). 'Optimal investment with costly reversibility', *Review of Economic Studies*, vol. 63, pp. 581–93.

Andersen, T. and Bollerslev, T. (1998). 'Answering the skeptics: yes, standard volatility models do provide accurate forecasts', *International Economic Review,* vol. 39, pp 885–905.

Arellano, M. and Bond, S. (1991). 'Some tests of specification for panel data: Monte Carlo evidence and an application to employment equations', *Review of Economic Studies*, vol. 58, pp 277–97.

Bertola, G. (1988). 'Irreversible investment', Massachusetts Institute of Technology doctoral thesis, re-printed in *Research in Economics,* vol. 52, pp. 3–37.

Bertola, G. and Caballero, R. (1994) 'Irreversibility and aggregate investment', *Review of Economic Studies,* vol. 61, pp 223–46.

Billingsley, P. (1968). *Probability and Measure*, 2nd edition, New York: John Wiley & Sons.

Bloom, N. (2000). 'The real options effects of uncertainty on investment and labour demand', IFS working paper no. W00/15.

Bloom, N. and Van Reenen, J. (2001). 'Real options, patents, productivity and market value: evidence from a panel of British firms', Institute for Fiscal Studies working paper W00/21, http://www.ifs.org.uk/workingpapers/wp0021.pdf.

Bloom, N., Bond, S., and Van Reenen, J. (2001). 'The dynamics of investment under uncertainty', IFS working paper W01/05.

Bloom, N., Griffith, R., and Van Reenen, J. (2000). 'Do R&D tax credits work? Evidence from an international panel of countries 1979–1998', forthcoming *Journal of Public Economics*.

Blundell, R. and Bond, S. (1998). 'Initial conditions and moment restrictions in dynamic panel data models', *Journal of Econometrics*, vol. 87, pp. 115–43.

Blundell, R. Griffith, R. and Windmeijer, F. (1998), 'Individual effects and dynamics in count data models', forthcoming in *Journal of Econometrics*, and IFS working paper W95/15.

Blundell, R., Griffith, R. and Van Reenen, J. (1995). 'Dynamic count data models of technological innovation', ECONOMIC JOURNAL, vol. 105, pp. 333–4.

Blundell, R., Griffith, R. and Van Reenen, J. (1999) 'Market structure and innovation: evidence from British manufacturing firms', *Review of Economic Studies*, vol. 66, pp. 529–54.

Bond, S. Harhoff, D. and Van Reenen, J. (1999). 'Investment, R&D and financial constraints in Britain and Germany', IFS Working Paper 99/5.

Bond, S. and Chennells, L. (2000). 'Institutional share ownership and dividend behaviour', IFS mimeo.

Bosworth, D., Greenhalgh, C. and Stoneman, P. (2000). 'Patents, trademarks and market value: evidence from British firms', University of Warwick, mimeo.

Bosworth, D., Greenhalgh, C. and Wharton, A. (2000). 'Intangible assets and the market valuation of UK companies: evidence from fixed effects models', Oxford Intellectual Property Research Centre, Working Paper no 2.

Bosworth, D., Greenhalgh, C. and Longland, M. (2001). 'Technological activity and employment in a panel of UK firms', forthcoming *Scottish Journal of Political Economy*.

Churchill, R. and Brown, J. (1987). *Fourier Series and Boundary Value Problems*. 4th edition, New York: McGraw Hill.

Cockburn, I. and Griliches, Z. (1988). 'Industry effects and appropriability measures in the stock markets valuation of R&D and patents', *American Economic Review*, vol. 78, pp. 419–23.

Dixit, A. (1989). 'Entry and exit decisions under uncertainty'. *Journal of Political Economy*, vol. 97, pp 620–38.

Dixit, A. and Pindyck, R. (1994). *Investment Under Uncertainty*, Princeton, New Jersey: Princeton University Press.

Eberly, J. and Van Mieghem, A. (1997). 'Multi-factor dynamic investment under uncertainty', *Journal of Economic Theory*, vol. 75, pp. 345–87.

Geroski, P. (1990). 'Innovation, technological opportunity and market structure', *Oxford Economic Papers*, vol. 42, pp. 586–602.

Griliches, Z. (1981). 'Market value, R&D and patents', *Economics Letters,* vol. 2, pp. 183–7.

Griliches, Z. (1990). 'Patent statistics as economic indicators: a survey', *Journal of Economic Literature*, vol.28, pp. 1661–707.

Griliches, Z., Hall, B. and Pakes, A. (1988) "R&D, patents, and market value revisited: is there a second (technological opportunity) factor?'', NBER Working Paper no. 2624.

Guiso, L. and Parigi, G. (1999). 'Investment and demand uncertainty', *Quarterly Journal of Economics*, vol. 114, pp. 185–227.

Hall, B., Jaffe, A. and Trajtenberg, M. (2000). 'Market value and patent citations: a first look', NBER Working Paper 7741.

Hausman, J., Hall, B. and Griliches, Z. (1984). 'Economic models for count data and an application to the patents–R&D relationship', *Econometrica*, 52, 909–38.

Henderson, R., Jaffe, A. and Trajtenberg, M. (1993). 'Geographic localization of knowledge spillovers as evidenced by patent citations', *Quarterly Journal of Economics*, vol. 105(3), pp. 577–94.

Jaffe, A. and Trajtenberg, M. (1998). 'International knowledge flows: evidence from patent citations', NBER Working Paper No. 6507.

Kreyszig, E. (1999). *Advance Engineering Mathematics*. New York, John Wiley & Sons.

Leahy, J. and Whited, T. (1998). 'The effects of uncertainty on investment:some stylized facts', *Journal of Money Credit and Banking*, vol. 28, pp. 64–83.

Lerner, J. (2000). '50 years of patent office practice', National Bureau of Economic Research Working Paper 7477.

McDonald, R. and Siegel, J. (1986). 'The Value of waiting to invest', *Quarterly Journal of Economics*, 101(4), 707–27.

Pakes, A. (1985). 'On patents, R&D, and the stock market rate of return', *Journal of Political Economy*, vol. 93, pp. 390–409.

Pakes, A. (1986). 'Patents as options: some estimates of the value of holding European patents stocks', *Econometrica*, vol. 54, pp. 755–84.

Pavitt, K., Robson, N. and Townsend, J. (1987). 'The size distribution of innovating firms in the UK: 1945–1983', *Journal of Industrial Economics*, vol 35, pp. 297–316.

Pindyck, R. (1988). 'Irreversible investment, capacity choice, and the valuation of the firm', *American Economic Review*, vol. 79, pp. 969–85.

Solow, R. (1957), 'Technical change and the aggregate production function', *Review of Economics and Statistics*, vol. 39, pp. 257–63.

Trajtenberg, M., Jaffe, A. and Hall, B. (2000). *NBER-Case-Western University Patents Data*. NBER, Brandeis University, UC Berkeley, and Tel Aviv University.